Quantum Computing for Markowitz Portfolio Optimization in Global Renewable Energy Stock Markets*

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Abstract. The financial sector embraces sustainability and green investment strategies, constructing optimal investment portfolios in the renewable energy sector requires computational methods capable of handling both high volatility and complex variable interactions. This study investigates and compares the performance of classical and quantum computing approaches in portfolio optimization using a set of leading global renewable energy stocks. The classical approach is based on the Mean-Variance Portfolio Optimization (MVPO), while the quantum approach employs two hybrid quantum-classical algorithms: the Quantum Approximate Optimization Algorithm (QAOA) and the Variational Quantum Eigen solver (VQE). The results reveal that quantum algorithms generate portfolios with significantly lower variance compared to the MVPO. This indicates superior risk diversification and improved risk-adjusted returns. The strength of quantum methods lies in their capacity to search for a broader solution space and avoid being trapped in local minimum, especially in environments characterized by uncertainty and noise. These advantages enable quantum techniques to identify more efficient portfolio allocations that better balance risk and return. This research provides empirical support for the growing potential of quantum computing in financial optimization. Despite current hardware limitations, hybrid quantum-classical algorithms show promising outcomes that could reshape decision-making processes in financial markets. The findings indicate that quantum-enhanced optimization could become a valuable tool for managing complex investment strategies, particularly in addressing the challenges posed by thin-tail and fat-tail phenomena in portfolio optimization.

Keywords: Quantum Computing, Markowitz Portfolio Optimization, Renewable Energy Stocks, Portfolio Optimization, Stock Market, Thin-tail, Fat-tail, Risk Management.

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1 Introduction

The growing devotion of world to sustainability is making renewable energy sources more important (IRENA, 2023). This spectacle shift has led to renewable energy companies and investors coming to the creation of stock exchanges towards clean energy boom. Figure 1 shows the amount of world's renewable energy generated from different sources in Terawatt-hours (TWh) for the periods between 1965 and 2022. Hydropower seems to have a steady growth over past six decades, reaching up to 4,000 TWh by 2022, represented by blue line on the figure. Wind and Solar power, represented by brown and red lines, display the fastest growing rates of all other sources in recent years. Even though hydropower remains a dominant renewable energy source at present, the recent rapid growing rate of wind and solar power suggests they might take over a leading role in future renewable energy markets.

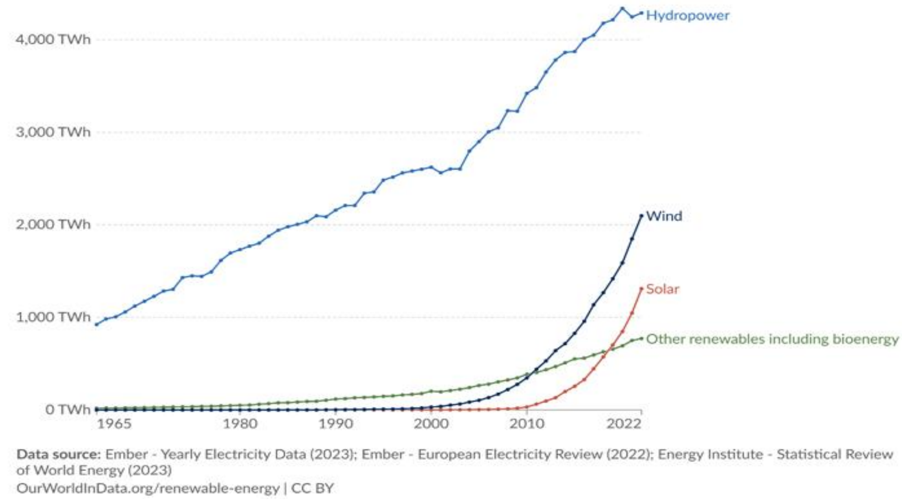

Fig. 1. Modern Renewable Energy Generation

a watt-hour is 1 Joule per second, so a watt-hour equals 3600 Joules of energy which are delivered by one watt of power for one hour; Kilowatt-hours (kWh), or a thousand-watt hours; Megawatt-hours (MWh), or a million watt-hours; Gigawatt-hours (GWh), or a billion watt-hours; & Terawatt-hours (TWh) or a trillion watt-hours. The growing attention on renewable energy has created a unique challenge faced by different investors deciding on the trade-off between the amount of risk and the return on capital assets. In terms of literature, a personal portfolio is a sum of financial investments including stocks, assets and other forms of possessions which give future returns. It is manageable by individuals or groups. Thus, the proper analysis of portfolios is essential in growing business or financial wellbeing of those who build portfolios. An investment portfolio that balances risk and return refers to Modern Portfolio Optimization (MPO). It is also one of the most common issues in the financial world for most investors with different levels of capital (Qu & Suganthan, 2011).

In 1952, Harry Markowitz came up with a revolutionary concept to evaluate portfolio called Mean-Variance (MV) model, introducing a modern version of portfolio theory. Since his milestone, a number of pioneering works also have been published analyzing present trends and future directions on portfolio optimization. Markowitz Mean-Variance model can be applied for both linear and quadratic relations in mathematical finance and significantly useful in portfolio construction. However, some of the existing related works believed that classical MPO techniques including Mean-Variance model do not suit well for renewable energy investments but the efficiency of quantum computing in complex portfolio problems (Zhao et al, 2022; Xin-gang et al, 2023; Buonaiuto et al, 2023).

Therefore, authors further conduct a study on modern portfolio optimization of world's top 20 renewable energy stocks as focus by applying both classical and quantum computing. Differently to most papers, authors explored the potential of both classical and quantum computing for MPO based on Markowitz mean-variance model. Authors included two distinct methods for multi-objective portfolios: the classical Mean-Variance model, and quantum Quadratic Unconstrained Binary Optimization (QUBO) model with VQE algorithm.

Our primary objectives are first to investigate how well classical and quantum techniques perform in optimizing portfolios of world's top renewable energy stocks, and second to compare the performance of classical and quantum approaches for renewable energy stock exchange portfolios.

2 Literature Reviews

In the field of portfolio optimization (MPO), (Markowitz, 1952) introduced the pioneering mean-variance model, which laid the foundation for modern portfolio theory (MPT). This model focuses on optimizing portfolio returns by balancing risk and return, making it an essential component of investment strategies. Building upon this foundational work, (Lim, K., 2015) applied the concept of Value at Risk (VaR) for risk management in MPO, providing a quantitative measure to assess potential losses in portfolio value under specific conditions. Further advancements in portfolio optimization were explored by (DeMiguel et al., 2009), who examined optimal portfolio selection in dynamic market environments. They emphasized the need to adjust portfolio strategies based on changing market conditions, highlighting the dynamic nature of financial markets and the complexities involved in portfolio management. In terms of algorithmic efficiency, classical algorithms analyzed for MPO, providing insights into the efficiency of traditional methods in portfolio optimization. Methodologically, (Ehrgott, 2008) contributed to the field by applying multicriteria optimization techniques for MPO using classical computing, broadening the scope of optimization beyond the traditional risk-return trade-off. Similarly, Rockafellar , 2013) advanced the understanding of optimal portfolio selection by incorporating conditional VaR into classical methods, offering a more nuanced approach to managing risk in portfolio selection. The integration of quantum computing into portfolio optimization emerged as a promising area of research. (Orús et al., 2019) comprehensive overview of how quantum computing can enhance financial problem-solving, particularly in portfolio optimization, risk analysis, and derivative pricing, highlighting its potential to transform computational finance in the future. In addition, (Huynh, L., 2023) provided a comprehensive survey of quantum machine learning applications in finance, including MPO, which highlighted the growing role of quantum techniques in enhancing portfolio management strategies. The shift towards quantum-enhanced methods for optimization reflects the increasing complexity of financial markets and the demand for more efficient and robust computational solutions.

In the context of renewable energy, (Bouzid, 2018) propose a Multi-Segment Fuzzy Goal Programming (MS-FGP) model to optimize renewable energy portfolios under uncertainty a study demonstrates that the MS-FGP model can effectively support decision-makers in selecting balanced and sustainable energy investment portfolios under uncertain conditions. adding a layer of sustainability considerations to the optimization process. Lastly, in the context of ASEAN exchanges, (Chaiboonsri, 2021) explored portfolio optimization based on quantum mechanics under risk management, emphasizing the role of quantum computing in managing risk. For this research we will study and compare the efficiency of using Quantum Computing and Classical Computing in Markowitz Portfolio Optimization in the global renewable energy stock market. The focus is on the application of Quantum Algorithms to improve computational capability and increase the efficiency of investment portfolio selection models. Compared to classical methods that have speed limitations and large data processing. This research will focus on examining the advantages and disadvantages of using each method to obtain accurate results and be able to apply them more effectively in the renewable energy market.

3 The conceptual framework and Methodology

3.1 Conceptual Framework

This study focuses on optimizing investment strategies for the world's top 20 renewable energy stocks by applying a combination of data analysis and advanced optimization techniques. The research employs a series of methods, starting with the preparation and screening of data, followed by clustering analysis to identify patterns, and concludes with portfolio optimization using both classical and quantum models. The aim is to create a more efficient and informed approach to managing renewable energy stock investments, particularly by minimizing risk while maximizing returns. The initial step involves screening the data for the 20 renewable energy stocks, ensuring that the dataset is robust and suitable for time-series analysis. This process includes conducting tests to assess the stationarity of the data and summarizing the essential characteristics of the stock returns, providing a foundational understanding of the data's behavior. Next, the study applies K-Means clustering, a machine learning technique, to group the stocks based on similarities in their return patterns. By dividing the stocks into distinct clusters, the analysis uncovers relationships between stocks that exhibit similar risk and return characteristics, which is crucial for effective portfolio construction. Following the clustering phase, the study employs Markowitz Portfolio Optimization, initially using Gaussian distribution to model stock returns. This approach seeks to determine the optimal allocation of stocks within the portfolio, balancing expected returns and risk. The classical Markowitz model is then extended by incorporating Quantum distribution, which leverages quantum computing techniques to enhance the accuracy and efficiency of portfolio optimization, particularly when dealing with large datasets and complex relationships.

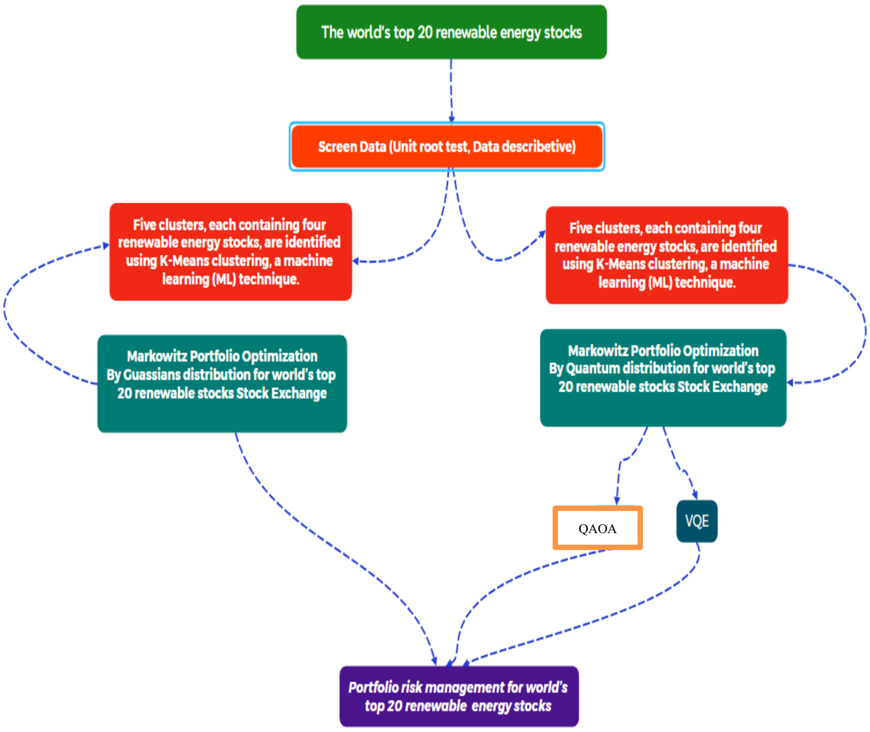

Fig. 2. Conceptual Framework of this Research

Finally, the study explores portfolio risk management through The Quantum Approximate Optimization Algorithm (QAOA) and Variational Quantum Eigen-solver (VQE), which are technique often used in quantum computing. This method helps optimize portfolio decisions by balancing the trade-off between risk and return, offering a more effective solution for managing complex portfolios.

3.2 Modern Portfolio Optimization

As described earlier in the introduction part, Modern Portfolio Theory is a strategy that helps investors find the best portfolios. It does this by either trying to get the highest possible return for a certain level of risk or trying to get the least amount of risk for an expected return. To achieve this, it follows three steps: first, it uses historical data to estimate the average return, risk and relationships between each pair of stocks, second, it calculates how much of each stock to hold in order to reduce the overall risk of portfolio while expecting for a targeted return and finally, it picks the best optimal portfolio by looking at which one offers the highest return and lowest risk. In fact, many people already assume that managing a collection of assets (N) for the investment is the basic problem in portfolio optimization.

For each stock, the expected return r_i^t between the day t – 1 and t can be calculated by following measuring formula below.

$$r_i^t = \frac{r_i^t - r_i^{t-1}}{\rho_i^{t-1}} \qquad (1)$$

Where $\rho_{i,t}$ is the stock price of the i^{th} stock at time t. Assuming a normal distribution of the returns, the average of stock prices at each time t on the set of historical observations is a good estimator of the expected return. Thus, the expected return of each stock/asset ω_i is calculated by:

$$\omega_i = E[r_i] = \frac{1}{\tau}\sum_{t=1}^{\tau} r_i^t \qquad (2)$$

Following this principle, the variance of each asset returns and covariance between returns of different assets over historical series can be calculated as follows:

$$\sigma_i^2 = E[(r_i - \omega_i)^2] \qquad (3)$$

$$\rho_{ij} = cov(r_i r_j)/(\sigma_i \sigma_j) \qquad (4)$$

Where $\omega_i = E(r_i)$ = expected value of the return; and $cov(r_i r_j) = \sigma_{ij} = E[(r_i - \omega_i)((r_i - \omega_i)]$. Multi-objective portfolio (MOP) optimizes expected return and volatility at the same time using the cost function based on the risk aversion parameter π (a trade-off between risk and return).

$$\frac{\text{Min}}{\omega} \quad \omega^T \Sigma \omega - \pi \omega^T r$$

$$Subject\ to\ \ \omega 1 = 1$$

$$\omega_i \geq 0, i = 1, \ldots, N$$

Similar to Markovitz's Mean Variance, portfolio's Maximum Return Portfolio (MRP) for a tarted volatility K can be written as follows.

$$\frac{Max}{\omega} \quad \omega^T r$$

$$Subject\ to\ \omega^T \Sigma \omega = K\ ;\ \ \omega 1 = 1$$

$$\omega_i \geq 0, i = 1, \ldots, N$$

3.2 Mean-Variance Portfolio Optimization model (MVPO)

Modern portfolio theory is believed to be launched by the mean-variance (MV) model introduced in 1952 by Harry Markowitz (Markowitz, 1952). This theory evaluates the portfolio return by specific level of risk to maximize the expected return. The MV model optimizes the portfolio's performance by balancing risk and return, and it is expressed mathematically in equation (1).

$$E(r_\rho) = \sum_{i=1}^{N} \omega_i E(r_i) \qquad (5)$$

In formula [1], $E(r_\rho)$ = portfolio's expected return; N = number of assets in the portfolio; ω_i=weight of asset i in portfolio; and $E(r_i)$= expected return of asset i.

$$\sigma_\rho^2 = \text{Var}\,(r_\rho) = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i \omega_j \sigma_{ij} = \omega^\tau \Phi \omega \qquad (6)$$

Where σ_ρ^2=variance of the portfolio; ω_i, ω_j = assets weights of y and x in the portfolio; and σ_{ij}= covariance of assets' returns of the portfolio. The covariance matrix of a portfolio constructed from N different assets can be calculated like;

$$Cov. = \Phi = \begin{bmatrix} r_{11} & \cdots & r_{1N} \\ \vdots & \ldots & \vdots \\ r_{N1} & \cdots & r_{NN} \end{bmatrix} \qquad (7)$$

The classical mean-variance portfolio optimization problem can be formulated as follows.

$$\begin{aligned} &\text{Minimize}\ (\omega) \quad \omega^\tau \Phi \omega \\ &\text{s.t} \quad \omega^\tau E\big((r_\rho\big) = r_{target}, \\ &\omega^\tau |1| = 1,\ \ \omega_{ij} \geq 0 \rightarrow i = 1, .., N \end{aligned}$$

3.3 Sharpe Ratio

The Sharpe ratio, first developed by William F. Sharpe in 1996 (Sharpe, 1996), is a metric used to evaluate the performance of investment by considering both its returns and level of risk involved. In general, it is the link between portfolio's excess target return and total standard deviation using the equation written in (8).

$$S_p = \frac{r_p - r_f}{\sigma_p} \qquad (8)$$

Where S_p = maximum excess return; r_p= portfolio's expected return; r_f= risk free rate; and σ_p= expected volatility or standard deviation of the portfolio's return.

3.4 The Quantum Approximate Optimization Algorithm (QAOA)

The standard QAOA framework minimizes the cost function over all investment options, including infeasible ones. (Brandhofer et al, 2023). To penalize constraint violations, a penalty term is added to FFF, yielding the following form:

$$F^{(A)}(z_1, \dots, z_n) = F(z_1, \dots, z_n) + A(\sum_{i=1}^{n} z_i - B)^2 \tag{9}$$

The penalty coefficient A must be selected carefully large enough so that all infeasible solutions (those violating the constraint) produce a penalized cost $F^{(A)}$greater than the minimum value F_{min} among feasible solutions. However, setting A too large can negatively affect the effectiveness of QAOA. Our findings indicate that using an excessively large value of A is unnecessary. The objective is not merely to distinguish between feasible and infeasible configurations, but rather to identify those that yield results close to F_{min} . Therefore, it is sufficient to choose A in a way that ensures an appropriate separation between optimal feasible and suboptimal infeasible states.

$$F_{min}^{(nf)} = \overset{min}{\sum_{i} z_i \neq B} F^{(A)}(z_1, \dots, z_n) \tag{10}$$

That is, the lowest value of the penalized cost function $F^{(A)}$ for all invalid solutions should remain sufficiently distant from the optimal value F_{min}. To achieve this, we apply the following approach to guarantee that separation.

$$F_{min}^{(nf)} \geq \frac{1}{2}(F_{min} + \bar{F}) \tag{11}$$

Where $\bar{\mathrm{F}}$ is the average score of all portfolios that meet the budget constraint:

$$\bar{\mathrm{F}} = \frac{1}{\binom{n}{B}} \sum_{z_1 + \dots + z_n = B} F(z_1, \dots, z_n) \tag{12}$$

F = Cost function that measures risk, negative return, or a combination of investment objectives.
z_1 = Whether or not to include stock *i* in the selected portfolio.
B = The budget constraint (e.g., total number of stocks or total capital limit).

3.5 Variational Quantum Eigensolver (VQE)

In 2014, Peruzzo et al first developed Variational Quantum Eigensolver (VQE) using variational principle to compute the ground state energy of a Hamiltonian which is a problem that is central to quantum chemistry and condensed physics. VQE is one of most popular hybrid quantum-classical algorithm, being introduced in NISQ quantum computers in practical applications (Tripathy et al, 2022). Applying VQE is usually to solve portfolio optimization problem needs to be in a quadratic program. Mostly researchers use Qiskit's finance application to convert the portfolio optimization problem (POP) into a quadratic program. This powerful hybrid quantum algorithm is to minimize an upper bound on the lowest expectation value of an observable with respect to a parameterized or ansatz wave-function (Tilly et al, 2022). For given a Hamiltonian $\hat{H}$ and an ansatz wave-function $|\psi(\theta) >$, the ground state energy E_0 is by.

$$E_0 \leq \frac{< \psi(\theta)|\hat{H}|\psi(\theta) >}{< \psi(\theta)|\psi(\theta) >}, \forall\ \theta \tag{11}$$

The goal of VQE is to find the optimal set of parameters θ corresponding to energy E_{min} for which $|E_{min} - E_0| < \epsilon$, ϵ being an arbitrary small constant. To perform this minimization on a quantum computer, the wavefunction $|\psi(\theta) >$ applied to an initial state for N qubits. On the quantum device, thus, the VQE optimization problem is formulated as follows:

$$E_{VQE} = \frac{min}{\theta} < 0|\, \cup^{\dagger}\, (\theta)\hat{H} \cup (\theta)|0 > \tag{12}$$

This above equation is also regarded as the cost function in the VQE framework. Where $\cup$ (θ) is parametrized unitary operator that gives the ansatz wave-function when applied on the initial state. The general form of Hamiltonian is expressed in equation (13) and equation (14) represents how it is mapped to spin operators which allows a direct measurement on a quantum computer. The base is formed by Pauli strings: $\widehat{\rho_\alpha} \in \{l, X, Y, Z\}^{\otimes N}$, with N qubits used to model wavefunction.

$$\hat{H} = \sum_{\alpha}^{\rho} \omega_\alpha \hat{\rho}_\alpha \tag{13}$$

$$E_{VQE} = \frac{min}{\theta} \sum_{\alpha}^{\rho} \omega_\alpha (0|\, \cup^{\dagger}\, (\theta)\hat{H} \cup (\theta)|0 > \tag{14}$$

In equation (13), ω_α= a set of weights; ρ=the number of Pauli strings in Hamiltonian, and it then gets as written in equation (14) where, each term $E_{\rho_\alpha} =< 0|\, \cup^{\dagger}\, (\theta)\hat{H} \cup (\theta)|0 >$ corresponds to the expected value of a Pauli string $\hat{\rho}_\alpha$, leading to a computable stage on a quantum device while $E_{VQE} = min_\theta \sum_\alpha \omega_\alpha E_{\rho_\alpha}$is computed on a classical computer.

4 Results and Discussion

4.1 Data description

This study analyzes the stock performance of 20 selected renewable energy companies over the period from 2022 to 2025. The companies include BE, BEP, BLDP, CSIQ, DQ, ENPH, DE, FSLR, JKS, NEE, NIO, ORS, PLUG, RUN, SHLS, SEDG, TSLA, VWS, SMR, and ARRY.

Daily stock prices and returns were collected from publicly available financial databases, with necessary adjustments made for dividends and stock splits to ensure consistency. The selected stocks represent a diverse range of companies engaged in renewable energy production, solar technology, electric vehicles, and clean energy infrastructure.

Table 1. Result analysis stock performance of 20 selected renewable energy companies

	ARRY	BE	BEP	BLDP	CSIQ	DE	DQ
Mean	-0.0008	0.0003	-0.0006	-0.0027	-0.0013	0.0001	-0.0011
Median	-0.0029	-0.0021	-0.0017	-0.0063	-0.0042	0.0001	-0.0042
Maximum	0.2177	0.4284	0.0850	0.1554	0.1464	0.0554	0.1979
Minimum	-0.2147	-0.2589	-0.0773	-0.1590	-0.1464	-0.0460	-0.2392
Variance	0.0026	0.0026	0.0004	0.0017	-0.0773	0.0001	0.0018
Jarque-Bera	145.6521	2246.984	80.27181	37.11902	37.82919	74.41367	146.2727
Probability	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
PP-unit root test	-26.6305	-27.9245	-25.1654	-28.2479	-28.5408	-27.1041	-28.8119
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Observations	**763**	**763**	**763**	**763**	**763**	**763**	**763**
	ENPH	**FSLR**	**JKS**	**NEE**	**NIO**	**ORS**	**PLUG**
Mean	-0.0011	0.0009	-0.0011	-0.0002	-0.0025	-0.0013	-0.0032
Median	-0.0019	-0.0013	-0.0024	-0.0004	-0.0041	-0.0006	-0.0054
Maximum	0.2488	0.1721	0.1721	0.0729	0.2690	0.2114	0.6240
Minimum	-0.2192	-0.1325	-0.1749	-0.0754	-0.1824	-0.2121	-0.5049
Variance	0.0019	0.0012	0.0018	0.0003	0.0025	0.0009	0.0042
Jarque-Bera	646.2583	145.0669	44.2971	278.3386	185.0005	2639.96	8218.059
Probability	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
PP-unit root test	-27.6498	-28.048	-27.8876	-26.0293	-29.0313	-29.0092	-28.419
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Observations	**763**	**763**	**763**	**763**	**763**	**763**	**763**
	RUN	**SEDG**	**SHLS**	**SMR**	**TSLA**	**VWS**	
Mean	-0.0016	-0.0039	-0.0020	0.0011	0.0002	-0.0008	
Median	-0.0035	-0.0054	-0.0034	0.0000	-0.0011	-0.0016	
Maximum	0.2712	0.1993	0.2583	0.3141	0.1464	0.1776	
Minimum	-0.1808	-0.4686	-0.1753	-0.4116	-0.1703	-0.2057	
Variance	0.0030	0.0027	0.0027	0.0040	0.0016	0.0008	
Jarque-Bera	85.04022	3587.087	238.5362	1130.617	63.00592	1243.032	
Probability	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	
PP-unit root test	-27.2935	-28.8291	-27.8459	-29.0257	-29.054	-28.3587	
Probability	0.000	0.000	0.000	0.000	0.000	0.000	
Observations	**763**	**763**	**763**	**763**	**763**	**763**	

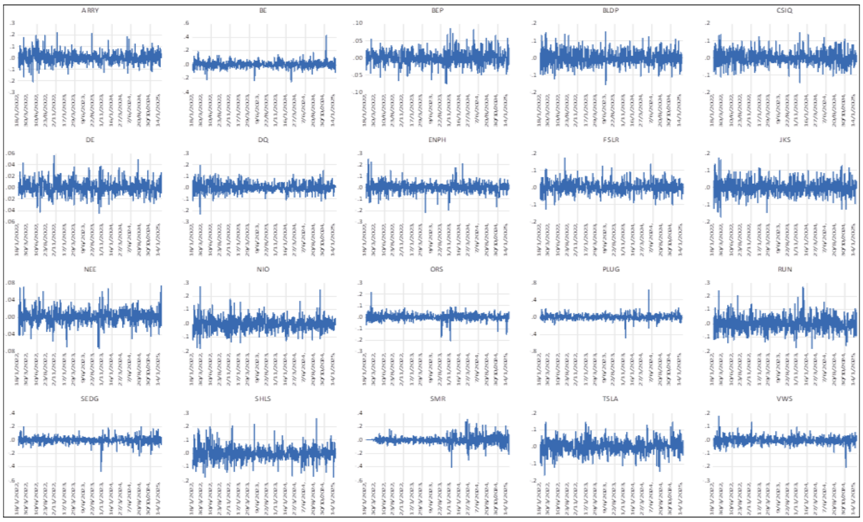

Fig. 3. Result Graphic stock performance of 20 selected renewable energy companies

4.2 K-Means clustering results

In the analysis of renewable energy stocks using K-Means Clustering, we have separated the stocks into 5 groups, with each group consisting of 4 stocks. This grouping allows us to see an overview of the relationships between the returns of stocks in different groups and enables deeper analysis of their impacts. The use of the K-Means technique helps to classify stocks with similar price movements into the same group, making the decision-making process for selecting stocks for portfolio investment easier and more efficient.

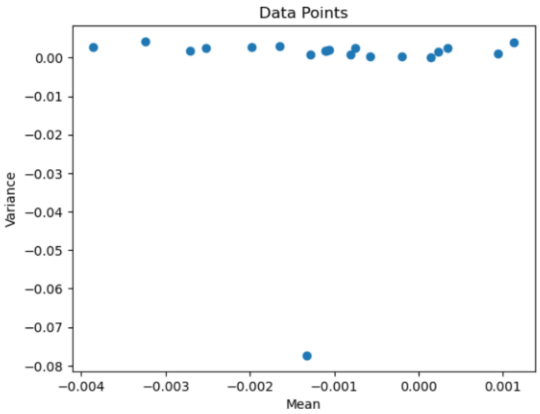

Fig. 4. before applying K-Means clustering analysis on renewable energy stock price Returns for step-1

Before applying K-Means clustering analysis on renewable energy stock price returns, this figure shows the distribution of stock returns for each stock in the renewable energy sector. The data points represent the return values for each stock, which have not yet been grouped by similarity. This visualization helps to understand the overall trend of stock returns and serves as a basis for comparison after clustering. The next step involves applying the K-Means clustering method, which groups the stocks based on their return patterns and volatility, allowing for better analysis and decision-making in portfolio optimization.

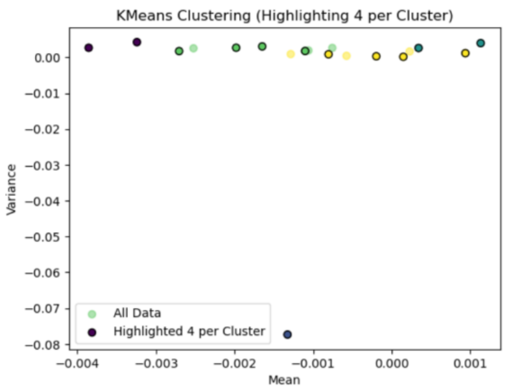

Fig. 5. after applying K-Means clustering analysis on renewable energy stock price Returns (1-step)

After applying K-Means clustering analysis on renewable energy stock price returns, this figure shows the resulting clusters of stocks based on their return patterns. The stocks have been grouped into 5 distinct clusters, each containing 4 stocks that exhibit similar return trends and volatilities. The visualization clearly highlights how the stocks have been categorized into different groups, allowing for more informed decision-making in portfolio construction and optimization. K-Means clustering helps to identify patterns that may not be immediately obvious from the raw data, providing a more structured approach to analyzing stock performance in the renewable energy sector.

Table 2. The result of applying K-Means clustering analysis on each renewable energy stock price returns for first group (1-step)

Points	Company	Mean	Variance	K-Means clustering
1	ARRY	-0.000755	0.002620754	1
2	BLDP	-0.002705	0.001722532	1
3	DQ	-0.001101	0.001754299	1
4	ENPH	-0.001059	0.001908549	1

The result of applying K-Means clustering analysis on the renewable energy stock price returns for the first group (1-step clustering) is shown in Table 2 The stocks in Group 1 include ARRY, BLDP, DQ, and ENPH. The average daily returns (Mean) for these stocks are all negative, suggesting a mild downward trend over the sample period. Specifically, the mean returns are 0.000755 for ARRY, –0.002705 for BLDP, –0.001101 for DQ, and –0.001059 for ENPH. In terms of volatility, measured by the variance, the stocks show moderate levels of risk, ranging from 0.001723 (BLDP) to 0.002621 (ARRY). These results suggest that stocks in Group 1 experienced similar return patterns with varying degrees of volatility during the analysis period.

Table 3. The result of applying K-Means clustering analysis on each renewable energy stock price returns for second group (1-step)

Points	Company	Mean	Variance	K-Means clustering
1	BEP	-0.000574	0.000412909	2
2	DE	0.000149	0.000146908	2
3	FSLR	0.000939	0.00115138	2
4	NEE	-0.000195	0.000286943	2

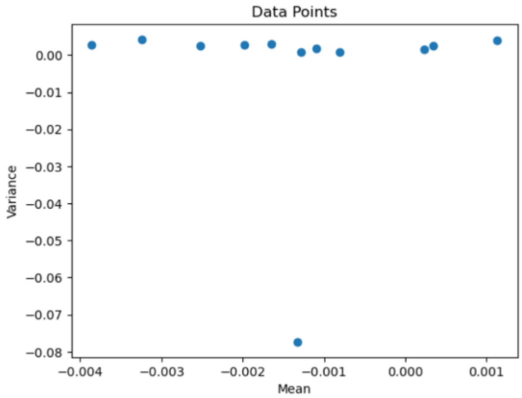

Fig. 6. before applying K-Means clustering analysis on renewable energy stock price returns for 2-step

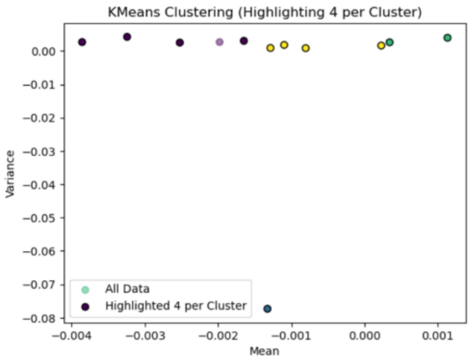

Fig. 7. after applying K-Means clustering analysis on renewable energy stock price Returns (2-step)

Table 4. The result of applying K-Means clustering analysis on each renewable energy stock price returns for third group (2-step)

Points	Company	Mean	Variance	K-Means clustering
1	NIO	-0.002522	0.002509922	3
2	PLUG	-0.003242	0.00422895	3
3	RUN	-0.001647	0.003032058	3
4	SEDG	-0.003856	0.002655588	3

The result of applying K-Means clustering analysis on the renewable energy stock price returns for the third group (2-step clustering) is shown in Table 4 Group 3 includes the stocks NIO, PLUG, RUN, and SEDG. See in Fig. 6 and Fig.7. The mean daily returns for all stocks in Group 3 are negative, indicating an overall downward trend during the analyzed period. Specifically, the mean returns are –0.002522 for NIO, –0.003242 for PLUG, –0.001647 for RUN, and –0.003856 for SEDG.

Regarding volatility, the stocks in this group exhibit relatively high variance values compared to Groups 1 and 2, ranging from 0.002656 (SEDG) to 0.004229 (PLUG). This suggests that Group 3 stocks experienced greater fluctuations in returns, reflecting higher risk levels in the renewable energy sector for this cluster.

Table 5. The result of applying K-Means clustering analysis on each renewable energy stock price returns for fourth group (2-step)

Points	Company	Mean	Variance	K-Means clustering
1	JKS	-0.001097	0.001802206	4
2	ORS	-0.001284	0.000894359	4
3	TSLA	0.000229	0.001623298	4
4	VWS	-0.000803	0.000840549	4

The table presents the result of applying K-Means clustering analysis (2-step approach) to the stock price returns of renewable energy companies in the fourth group, which includes JKS, ORS, TSLA, and VWS. For each company, the means and variance of the returns are reported along with the assigned cluster. JKS shows a mean return of -0.001097 and a variance of 0.001802206, while ORS has a mean return of -0.001284 and a variance of 0.000894359. TSLA yields a slightly positive mean return of 0.000229 with a variance of 0.001623298. Meanwhile, VWS records a mean return of -0.000803 and a variance of 0.000840549. Based on the K-Means clustering results, all four companies were grouped into the same cluster, identified as Cluster 4. See in Table 5

Table 6. The result of applying K-Means clustering analysis on each of renewable energy stock price returns for fifth group (2-step)

Points	Company	Mean	Variance	K-Means clustering
1	BE	0.000343	0.002596374	5
2	CSIQ	-0.001325	-0.077345	5
3	SHLS	-0.001979	0.002664889	5
4	SMR	0.001134	0.003951431	5

The table illustrates the outcomes of K-Means clustering analysis (2-step method) applied to the stock price returns of renewable energy companies in the fifth group, consisting of BE, CSIQ, SHLS, and SMR. Each company's mean and variance of stock returns are listed alongside their assigned cluster. BE exhibits a mean return of 0.000343 with a variance of 0.002596374. CSIQ has a negative mean return of -0.001325 and reports an anomalously negative variance value of -0.077345. SHLS shows a mean return of -0.001979 and a variance of 0.002664889, while SMR records a mean return of 0.001134 with the highest variance among the group, at 0.003951431. According to the clustering results, all four companies were classified into Cluster 5. See in Table 6

5.3 Markowitz Portfolio Optimization (Classical optimization)

In this section, the results of the classical Markowitz portfolio optimization for the first group of renewable energy stocks are presented. The optimization was conducted based on minimizing portfolio risk (variance) while achieving an acceptable expected return. Table 7 shows the optimization outcomes. Among all possible combinations of stock selections, the optimal portfolio configuration was identified as selecting the third and fourth stocks (DQ and ENPH), corresponding to a selection vector of [0, 0, 1, 1]. This portfolio achieved the lowest variance value of 0.0026.

Table 7. Result Markowitz Portfolio Optimization (Classical optimization) for First group

Optimal: selection [0. 0. 1. 1.], value 0.0026		
---------------- Full result --------------------		
selection	value	probability

[0 0 1 1]	**0.0026**	**1.0000**
[1 1 1 1]	4.0403	0.0000
[0 1 1 1]	1.0137	0.0000
[1 0 1 1]	1.0128	0.0000
[1 1 0 1]	1.0143	0.0000
[0 1 0 1]	0.0041	0.0000
[1 0 0 1]	0.0031	0.0000
[0 0 0 1]	1.0093	0.0000
[1 1 1 0]	1.0148	0.0000
[0 1 1 0]	0.0044	0.0000
[1 0 1 0]	0.0037	0.0000
[0 0 1 0]	1.0097	0.0000
[1 1 0 0]	0.0050	0.0000
[0 1 0 0]	1.0110	0.0000
[1 0 0 0]	1.0102	0.0000
[0 0 0 0]	4.0326	0.0000

The probability of the optimal selection [0, 0, 1, 1] was found to be 1.0000, indicating absolute certainty under the given optimization setup. Other combinations, although evaluated, resulted in higher portfolio variances and probabilities close to zero. These results suggest that, for the first group, including only DQ and ENPH in the portfolio provides the most efficient risk-return balance among the options considered.

Table 8. Result Markowitz Portfolio Optimization (Classical optimization) for Second group

Optimal: selection [0. 1. 1. 0.], value -0.0005		
----------------- Full result ---------------------		
selection	value	probability

[0 1 1 0]	**-0.0005**	**1.0000**
[1 1 1 1]	4.0187	0.0000
[0 1 1 1]	1.0040	0.0000
[1 0 1 1]	1.0058	0.0000
[0 0 1 1]	-0.0004	0.0000
[1 1 0 1]	1.0064	0.0000
[0 1 0 1]	0.0003	0.0000
[1 0 0 1]	0.0020	0.0000
[0 0 0 1]	1.0046	0.0000
[1 1 1 0]	1.0058	0.0000
[1 0 1 0]	0.0014	0.0000
[0 0 1 0]	1.0037	0.0000
[1 1 0 0]	0.0019	0.0000
[0 1 0 0]	1.0043	0.0000
[1 0 0 0]	1.0061	0.0000
[0 0 0 0]	4.0171	0.0000

The optimal portfolio selection for the second group of renewable energy stocks is obtained using the classical Markowitz Portfolio Optimization approach. The selected portfolio is represented by the combination of stocks [0, 1, 1, 0], where the 1's correspond to the chosen stocks and the 0's indicate the non-selected ones. The optimal value (expected return) of this portfolio is –0.0005, suggesting a slight negative return. The optimal portfolio selection of [0, 1, 1, 0] indicates that the most suitable stocks to include in the portfolio are DE and FSLR, which are assigned to positions 2 and 3, respectively, while the remaining stocks (BEP and NEE) are excluded. The variance in the value of different portfolio selections is quite high, with the best selection (portfolio [0, 1, 1, 0]) showing a slightly negative return of -0.0005. In this analysis, the optimal portfolio is the one that minimizes risk while achieving the desired return. The probability for the optimal selection [0, 1, 1, 0] with a value of –0.0005 is 1.0000, meaning it is the most likely combination among the alternatives considered.

Table 9. Result Markowitz Portfolio Optimization (Classical optimization) for Third group

Optimal: selection [0. 1. 1. 0.], value 0.0055		
----------------- Full result ---------------------		
selection	value	probability

[0 1 1 0]	**0.0055**	**1.0000**
[1 1 1 1]	4.0680	0.0000
[0 1 1 1]	1.0230	0.0000
[1 0 1 1]	1.0234	0.0000
[0 0 1 1]	0.0059	0.0000
[1 1 0 1]	1.0248	0.0000
[0 1 0 1]	0.0074	0.0000
[1 0 0 1]	0.0076	0.0000
[0 0 0 1]	1.0176	0.0000
[1 1 1 0]	1.0231	0.0000
[1 0 1 0]	0.0060	0.0000
[0 0 1 0]	1.0157	0.0000
[1 1 0 0]	0.0072	0.0000
[0 1 0 0]	1.0171	0.0000
[1 0 0 0]	1.0175	0.0000
[0 0 0 0]	4.0547	0.0000

The analysis applied K-Means clustering and a subsequent selection optimization process on the third group of renewable energy stocks, which included NIO, PLUG, RUN, and SEDG. The optimal portfolio selection from this group was [0, 1, 1, 0], indicating that PLUG and RUN were selected, while NIO and SEDG were excluded. This configuration yielded the lowest objective function value of 0.0055, which suggests that this combination provides the best risk-return profile among all considered subsets. The probability associated with this optimal selection was 1.0000, showing strong certainty and robustness in the model's recommendation. All other combinations, including full inclusion or other partial selections, resulted in significantly higher objective values and effectively zero probabilities, reinforcing the reliability of the selected subset. These findings indicate that PLUG and RUN demonstrate better performance or diversification benefits within the third group, making them favorable choices for constructing an efficient renewable energy investment portfolio.

Table 10. Result Markowitz Portfolio Optimization (Classical optimization) for Fourth group

Optimal: selection [0. 0. 1. 1.], value 0.0009		
---------------- Full result --------------------		
selection	value	probability
--		
[0 0 1 1]	**0.0009**	**1.0000**
[1 1 1 1]	4.0283	0.0000
[0 1 1 1]	1.0083	0.0000
[1 0 1 1]	1.0092	0.0000
[1 1 0 1]	1.0105	0.0000
[0 1 0 1]	0.0024	0.0000
[1 0 0 1]	0.0032	0.0000
[0 0 0 1]	1.0067	0.0000
[1 1 1 0]	1.0100	0.0000
[0 1 1 0]	0.0016	0.0000
[1 0 1 0]	0.0027	0.0000
[0 0 1 0]	1.0060	0.0000
[1 1 0 0]	0.0039	0.0000
[0 1 0 0]	1.0072	0.0000
[1 0 0 0]	1.0082	0.0000
[0 0 0 0]	4.0233	0.0000

In the analysis of the fourth group of renewable energy stocks, which included JKS, ORS, TSLA, and VWS, a two-step selection optimization was applied following K-Means clustering. The optimal portfolio selection was found to be [0, 0, 1, 1], indicating that TSLA and VWS were included in the portfolio, while JKS and ORS were excluded. This selection resulted in the lowest objective function value of 0.0009, which reflects the most favorable risk-return trade-off among all combinations considered. The probability for this optimal selection was 1.0000, confirming the model's confidence and the robustness of this outcome. All other portfolio combinations yielded significantly higher objective values and had a probability of nearly zero, further supporting the effectiveness of the selected subset. These findings suggest that TSLA and VWS possess superior performance characteristics or complement each other in risk diversification within this group, making them optimal candidates for constructing a well-balanced investment portfolio in the renewable energy sector.

Table 11. Result Markowitz Portfolio Optimization (Classical optimization) for Fifth group

Optimal: selection [1. 0. 0. 1.], value -0.0002		
----------------- Full result ---------------------		
selection	value	probability

[1 0 0 1]	**-0.0002**	**1.0000**
[1 1 1 1]	4.0326	0.0000
[0 1 1 1]	1.0100	0.0000
[1 0 1 1]	1.0094	0.0000
[0 0 1 1]	0.0012	0.0000
[1 1 0 1]	1.0086	0.0000
[0 1 0 1]	0.0005	0.0000
[0 0 0 1]	1.0062	0.0000
[1 1 1 0]	1.0122	0.0000
[0 1 1 0]	0.0039	0.0000
[1 0 1 0]	0.0034	0.0000
[0 0 1 0]	1.0096	0.0000
[1 1 0 0]	0.0025	0.0000
[0 1 0 0]	1.0087	0.0000
[1 0 0 0]	1.0081	0.0000
[0 0 0 0]	4.0288	0.0000

In the analysis of the fifth group of renewable energy stocks, which included BE, CSIQ, SHLS, and SMR, a two-step portfolio selection was performed following the application of K-Means clustering. The optimal portfolio was identified as [1, 0, 0, 1], indicating the inclusion of BE and SMR in the selected portfolio, while CSIQ and SHLS were excluded. This portfolio configuration achieved the lowest and most optimal objective function value of -0.0002, indicating a slightly negative risk-return value, which suggests the potential for a net risk-adjusted return or an efficient diversification effect. The probability associated with this optimal selection was 1.0000, indicating full model certainty. All other combinations yielded higher objective function values and effectively zero probability, supporting the robustness of the chosen selection. These findings imply that BE and SMR, when combined, offer a more desirable trade-off between return and risk or complement each other effectively in terms of portfolio diversification within this group of stocks.

5.4 Markowitz Portfolio Optimization (Classical mixed with Quantum optimization)

To explore potential enhancements in portfolio optimization, a hybrid quantum-classical method was applied to the first group of renewable energy stocks. Specifically, the Variational Quantum Eigen solver (VQE) algorithm was employed to solve the optimization problem. Table 12 presents the optimization outcomes. The optimal solution identified by VQE corresponds to selecting the third and fourth stocks (DQ and ENPH) with a selection vector of [0, 0, 1, 1], achieving the lowest variance value of 0.0026. However, the highest probability solution identified by VQE was [1, 0, 0, 1], corresponding to a slightly higher variance value of 0.0031 and a probability of 0.7513.

Table 12. Result Variational Quantum Eigen-solver (VQE) (Hybrid quantum-classical algorithm) for First group

Optimal: selection [0. 0. 1. 1.], value 0.0026		
---------------- Full result --------------------		
selection	value	probability
--		
[1 0 0 1]	**0.0031**	**0.7513**
[0 0 1 1]	**0.0026**	**0.1303**
[1 1 0 0]	**0.0050**	**0.0960**
[0 1 1 0]	**0.0044**	**0.0221**
[0 1 0 0]	1.0110	0.0001
[1 1 1 0]	1.0148	0.0001
[0 1 1 1]	1.0137	0.0000
[0 0 1 0]	1.0097	0.0000
[1 0 1 0]	0.0037	0.0000
[0 0 0 1]	1.0093	0.0000
[1 0 1 1]	1.0128	0.0000
[1 0 0 0]	1.0102	0.0000
[1 1 0 1]	1.0143	0.0000
[0 1 0 1]	0.0041	0.0000
[0 0 0 0]	4.0326	0.0000
[1 1 1 1]	4.0403	0.0000

Notably, the true optimal solution [0, 0, 1, 1] was assigned a probability of 0.1303 under the VQE execution. Other portfolio configurations showed higher variances and much lower probabilities, indicating that the quantum optimizer was able to approximate the classical optimum but exhibited some deviation in probability distribution. These findings demonstrate that while hybrid quantum-classical methods like VQE are promising, particularly for approximating near-optimal solutions, classical methods still maintain a slight advantage in precision for small-scale portfolio optimization problems.

Table 13. The Quantum Approximate Optimization Algorithm (QAOA)(Hybrid quantum-classical algorithm) of First Group

Optimal: selection [0. 0. 1. 1.], value 0.0026		
----------------- Full result ---------------------		
selection	value	probability
--		
[0 0 1 1]	**0.0026**	**0.1670**
[1 0 0 1]	**0.0031**	**0.1668**
[1 0 1 0]	**0.0037**	**0.1667**
[0 1 0 1]	**0.0041**	**0.1666**
[0 1 1 0]	**0.0044**	**0.1665**
[1 1 0 0]	**0.0050**	**0.1663**
[1 1 1 1]	4.0403	0.0000
[0 0 0 0]	4.0326	0.0000
[0 0 0 1]	1.0093	0.0000
[1 1 1 0]	1.0148	0.0000
[1 0 0 0]	1.0102	0.0000
[1 1 0 1]	1.0143	0.0000
[0 0 1 0]	1.0097	0.0000
[0 1 1 1]	1.0137	0.0000
[0 1 0 0]	1.0110	0.0000
[1 0 1 1]	1.0128	0.0000

In the context of applying hybrid quantum-classical algorithms, the Quantum Approximate Optimization Algorithm (QAOA) was utilized for portfolio optimization in the first group of renewable energy stocks. The QAOA method produced a near-optimal portfolio with a selection vector [0, 0, 1, 1], leading to a portfolio variance of 0.0026. As shown in Table X, QAOA's full result reveals that the optimal selection was assigned a probability of 0.1670. Additionally, the portfolio configurations with the next highest probabilities include [1, 0, 0, 1] with a value of 0.0031 (probability: 0.1668) and [1, 0, 1, 0] with a value of 0.0037 (probability: 0.1667). The selection probabilities for all configurations are relatively close to each other, indicating that QAOA explores a wide range of possible portfolio combinations. Like the VQE results, the highest probability portfolios correspond to configurations where stocks 3 and 4 (DQ and ENPH) are selected. However, unlike VQE, the selection probabilities for the top configurations in QAOA are more evenly distributed, highlighting the quantum algorithm's ability to search for the solution space in a less deterministic manner. These findings suggest that the QAOA algorithm, while also producing close-to-optimal solutions, offers a slightly more probabilistic approach compared to VQE, which may be valuable in contexts requiring diverse portfolio compositions.

Table 14. Result Variational Quantum Eigen-solver (VQE) (Hybrid quantum-classical algorithm) for Second group

Optimal: selection [0. 1. 1. 0.], value -0.0005		
----------------- Full result ---------------------		
selection	value	probability
--		
[1 0 0 1]	**0.0020**	**0.7443**
[0 0 1 1]	**-0.0004**	**0.1518**
[1 1 0 0]	**0.0019**	**0.0822**
[0 1 1 0]	**-0.0005**	**0.0215**
[0 1 0 0]	1.0043	0.0001
[1 1 1 0]	1.0058	0.0001
[1 0 1 0]	0.0014	0.0000
[0 0 1 0]	1.0037	0.0000
[0 1 1 1]	1.0040	0.0000
[1 0 1 1]	1.0058	0.0000
[0 0 0 1]	1.0046	0.0000
[1 0 0 0]	1.0061	0.0000
[1 1 0 1]	1.0064	0.0000
[1 1 1 1]	4.0187	0.0000
[0 1 0 1]	0.0003	0.0000
[0 0 0 0]	4.0171	0.0000

In the second group of renewable energy stocks comprising BEP, DE, FSLR, and NEE—the Variational Quantum Eigensolver (VQE), a hybrid quantum-classical algorithm, was employed to identify the optimal portfolio configuration based on quantum measurements. The result shows that the most probable configuration is [1, 0, 0, 1], which corresponds to the inclusion of BEP and NEE. This selection yielded the highest probability of 74.43% and an objective value of 0.0020, indicating that this combination is statistically the most stable and preferred under the VQE optimization framework. The second most probable configuration is [0, 0, 1, 1], which selects FSLR and NEE, with a probability of 15.18% and an objective value of -0.0004. While it appears relatively frequently, its lower objective value suggests that it may not perform as well as the most probable configuration in practical settings. Another configuration, [1, 1, 0, 0] (BEP and DE), appeared with 8.22% probability and a slightly better value of 0.0019, reflecting potential but less certainty under repeated quantum samples. the configuration with the lowest objective value, [0, 1, 1, 0], which includes DE and FSLR, was also flagged as the optimal solution based solely on the lowest energy (value = -0.0005). However, it occurred with only 2.15% probability, suggesting that although this configuration theoretically offer the lowest cost or energy under the VQE model, it is significantly less stable and less likely to emerge under repeated measurements. This further highlights the trade-off between theoretical optimality and practical stability within quantum optimization landscapes.

Table 15. The Quantum Approximate Optimization Algorithm (QAOA) (Hybrid quantum-classical algorithm) of Second Group

selection	value	probability
Optimal: selection [0. 1. 1. 0.], value -0.0005		
----------------- Full result ---------------------		
[1 0 0 1]	**0.0020**	**0.1675**
[1 1 0 0]	**0.0019**	**0.1674**
[1 0 1 0]	**0.0014**	**0.1670**
[0 1 0 1]	**0.0003**	**0.1662**
[0 0 1 1]	**-0.0004**	**0.1659**
[0 1 1 0]	**-0.0005**	**0.1658**
[0 1 1 1]	1.0040	0.0000
[1 0 0 0]	1.0061	0.0000
[1 1 1 1]	4.0187	0.0000
[1 1 1 0]	1.0058	0.0000
[1 0 1 1]	1.0058	0.0000
[1 1 0 1]	1.0064	0.0000
[0 0 0 0]	4.0171	0.0000
[0 0 1 0]	1.0037	0.0000
[0 0 0 1]	1.0046	0.0000
[0 1 0 0]	1.0043	0.0000

In the second group of renewable energy stocks—comprising BEP, DE, FSLR, and NEE—the Quantum Approximate Optimization Algorithm (QAOA), a hybrid quantum-classical algorithm, was employed to determine the optimal portfolio configuration. This method explores the solution space using a parameterized quantum circuit in conjunction with classical optimization techniques. According to the results, the configuration [0, 1, 1, 0], which includes the selection of DE and FSLR, was identified as the optimal solution with the lowest objective value of –0.0005 and a probability of 16.58%. This suggests that, under QAOA's framework, the combination of DE and FSLR represents the most favorable configuration in terms of minimizing the cost function. Several other configurations exhibited nearly identical probabilities, indicating that the QAOA algorithm yielded a relatively flat distribution across multiple low-cost solutions. For instance, [1, 0, 0, 1] (BEP and NEE) had a value of 0.0020 with a probability of 16.75%, [1, 1, 0, 0] (BEP and DE) had a value of 0.0019 (16.74%), and [1, 0, 1, 0] (BEP and FSLR) recorded 0.0014 (16.70%). Other similarly likely combinations included [0, 1, 0, 1] (DE and NEE) with a value of 0.0003 (16.62%) and [0, 0, 1, 1] (FSLR and NEE) with a slightly negative value of –0.0004 (16.59%). These results highlight the nuanced landscape of the quantum optimization process under QAOA, where multiple configurations exhibit close objective values and comparable probabilities. While the configuration [0, 1, 1, 0] technically offers the lowest cost, its advantage is marginal, and the solution space is populated by a set of near-optimal portfolios that may all be viable in practical implementations depending on external constraints or investor preferences.

Table 16. Result Variational Quantum Eigen-solver (VQE) (Hybrid quantum-classical algorithm) for Third group

Optimal: selection [0. 1. 1. 0.], value 0.0055		
----------------- Full result ---------------------		
selection	value	probability

[1 0 0 1]	**0.0076**	**0.6941**
[0 0 1 1]	**0.0059**	**0.1504**
[1 1 0 0]	**0.0072**	**0.1209**
[0 1 1 0]	**0.0055**	**0.0345**
[1 0 1 0]	0.0060	0.0001
[0 1 0 0]	1.0171	0.0000
[0 1 1 1]	1.0230	0.0000
[1 1 1 0]	1.0231	0.0000
[0 0 1 0]	1.0157	0.0000
[1 1 0 1]	1.0248	0.0000
[0 0 0 1]	1.0176	0.0000
[1 1 1 1]	4.0680	0.0000
[0 1 0 1]	0.0074	0.0000
[1 0 0 0]	1.0175	0.0000
[0 0 0 0]	4.0547	0.0000
[1 0 1 1]	1.0234	0.0000

For the third group, which includes NIO, PLUG, RUN, and SEDG, the analysis was performed using the Variational Quantum Eigen-solver (VQE) algorithm, which is a quantum-classical hybrid algorithm. The results of running the model are shown in the table below, which consists of three columns: stock selection pattern (selection), objective function value (value), and the probability resulting from random measurement in the quantum system. (probability) The table shows that the [1, 0, 0, 1] option (selecting NIO and SEDG stocks) gives an objective function value of 0.0076 and the highest probability of 0.6941, while the [0, 0, 1, 1] option (selecting RUN and SEDG stocks) gives a value of 0.0059 and a probability of 0.1504. The [1, 1, 0, 0] option (NIO and PLUG stocks) gives a value of 0.0072 and a probability of 0.1209, while the [0, 1, 1, 0] option (PLUG and RUN stocks) gives the lowest function value (0.0055) with a lower probability of 0.0345.

Table 17. The Quantum Approximate Optimization Algorithm (QAOA) (Hybrid quantum-classical algorithm) of Third Group

Optimal: selection [0. 1. 1. 0.], value 0.0055		
----------------- Full result ---------------------		
selection	value	probability
--		
[0 1 1 0]	**0.0055**	**0.1664**
[1 0 1 0]	**0.0060**	**0.1661**
[0 0 1 1]	**0.0059**	**0.1661**
[0 1 0 1]	**0.0074**	**0.1649**
[1 1 0 0]	**0.0072**	**0.1649**
[1 0 0 1]	**0.0076**	**0.1646**
[1 0 0 0]	1.0175	0.0008
[0 1 0 0]	1.0171	0.0008
[0 0 0 1]	1.0176	0.0008
[0 0 1 0]	1.0157	0.0008
[0 1 1 1]	1.0230	0.0008
[1 1 0 1]	1.0248	0.0008
[1 0 1 1]	1.0234	0.0008
[1 1 1 0]	1.0231	0.0008
[1 1 1 1]	4.0680	0.0005
[0 0 0 0]	4.0547	0.0002

The Quantum Approximate Optimization Algorithm (QAOA), a hybrid quantum-classical algorithm, was applied to the third group of companies, which includes NIO, PLUG, RUN, and SEDG. The aim was to explore the solution space and observe the distribution of selection configurations with respect to the objective function values and their corresponding probabilities. The result is summarized in the table below. The output indicates that the selection [0, 1, 1, 0], representing the inclusion of PLUG and RUN, yields an objective function value of 0.0055 with the highest probability among all configurations at 0.1664. Other prominent configurations include [1, 0, 1, 0] (NIO and RUN) with a value of 0.0060 and probability 0.1661, [0, 0, 1, 1] (RUN and SEDG) with a value of 0.0059 and probability 0.1661, [0, 1, 0, 1] (PLUG and SEDG) with a value of 0.0074 and probability 0.1649, and [1, 1, 0, 0] (NIO and PLUG) with a value of 0.0072 and probability 0.1649. These top selections share similar levels of probability, ranging from 0.1646 to 0.1664. In contrast, several configurations result in higher objective function values (above 1.0), such as [1, 0, 0, 0], [0, 1, 0, 0], [0, 0, 1, 0], and [1, 1, 1, 0], each with a very low probability of 0.0008. The configurations representing selection of all assets ([1, 1, 1, 1]) and none ([0, 0, 0]) yield the highest objective values of 4.0680 and 4.0547, respectively, and their corresponding probabilities are minimal at 0.0005 and 0.0002.

Table 18. Result Variational Quantum Eigen-solver (VQE) (Hybrid quantum-classical algorithm) for Fourth group

Optimal: selection [0. 0. 1. 1.], value 0.0009		
----------------- Full result ---------------------		
selection	value	probability
--		
[1 0 0 1]	**0.0032**	**0.7412**
[0 0 1 1]	**0.0009**	**0.1438**
[1 1 0 0]	**0.0039**	**0.0929**
[0 1 1 0]	**0.0016**	**0.0220**
[0 1 0 0]	1.0072	0.0000
[1 1 1 0]	1.0100	0.0000
[0 0 1 0]	1.0060	0.0000
[0 1 1 1]	1.0083	0.0000
[1 0 1 1]	1.0092	0.0000
[1 0 1 0]	0.0027	0.0000
[0 0 0 1]	1.0067	0.0000
[1 0 0 0]	1.0082	0.0000
[1 1 0 1]	1.0105	0.0000
[0 0 0 0]	4.0233	0.0000
[1 1 1 1]	4.0283	0.0000
[0 1 0 1]	0.0024	0.0000

The table presents the output from the Variational Quantum Eigen-solver (VQE) applied to a fourth group consisting of JKS, ORS, TSLA, and VWS. Each row in the table represents a binary selection of the assets, along with its corresponding objective function value and the probability of occurrence from the quantum circuit sampling. The selection [1, 0, 0, 1], which includes JKS and VWS, shows an objective value of 0.0032 with the highest sampling probability of 0.7412. The next entry, [0, 0, 1, 1], representing the selection of TSLA and VWS, has a lower objective value of 0.0009 and a probability of 0.1438. Following that, the configuration [1, 1, 0, 0] (JKS and ORS) results in a value of 0.0039 and appears with a probability of 0.0929. The combination [0, 1, 1, 0], including ORS and TSLA, yields a value of 0.0016 and a probability of 0.0220.

Table 19. The Quantum Approximate Optimization Algorithm (QAOA) (Hybrid quantum-classical algorithm) of Fourth Group

Optimal: selection [0. 0. 1. 1.], value 0.0009		
----------------- Full result ---------------------		
selection	value	probability

[0 0 1 1]	**0.0009**	**0.1670**
[0 1 1 0]	**0.0016**	**0.1668**
[0 1 0 1]	**0.0024**	**0.1667**
[1 0 1 0]	**0.0027**	**0.1667**
[1 0 0 1]	**0.0032**	**0.1665**
[1 1 0 0]	**0.0039**	**0.1663**
[1 1 1 1]	4.0283	0.0000
[1 1 0 1]	1.0105	0.0000
[0 0 0 0]	4.0233	0.0000
[0 0 1 0]	1.0060	0.0000
[1 1 1 0]	1.0100	0.0000
[0 0 0 1]	1.0067	0.0000
[1 0 1 1]	1.0092	0.0000
[0 1 0 0]	1.0072	0.0000
[1 0 0 0]	1.0082	0.0000
[0 1 1 1]	1.0083	0.0000

The table summarizes the results obtained from the Quantum Approximate Optimization Algorithm (QAOA), a hybrid quantum-classical optimization technique, applied to a four-asset portfolio. Each row represents a particular binary selection of assets, along with the corresponding objective function value and the probability observed from quantum sampling. The selection [0, 0, 1, 1], which corresponds to including TSLA and VWS in the portfolio, has the lowest objective value of 0.0009 and was sampled with a probability of 0.1670, making it the optimal solution identified by the algorithm. The next five selections also yielded relatively low objective values with similar probabilities. For instance, [0, 1, 1, 0], representing ORS and TSLA, had a value of 0.0016 and probability 0.1668. The selection [0, 1, 0, 1] (ORS and VWS) resulted in a value of 0.0024 and probability 0.1667, while [1, 0, 1, 0] (JKS and TSLA) had a value of 0.0027 and probability 0.1667. The combination [1, 0, 0, 1] (JKS and VWS) showed a slightly higher value of 0.0032 with a probability of 0.1665, followed by [1, 1, 0, 0] (JKS and ORS) with a value of 0.0039 and a probability of 0.1663.

Table 20. Result Variational Quantum Eigen-solver (VQE) (Hybrid quantum-classical algorithm) for Fifth group

Optimal: selection [1. 0. 0. 1.], value -0.0002		
----------------- Full result ---------------------		
selection	value	probability

[1 0 0 1]	**-0.0002**	**0.8099**
[0 0 1 1]	**0.0012**	**0.0968**
[1 1 0 0]	**0.0025**	**0.0772**
[0 1 1 0]	**0.0039**	**0.0157**
[0 1 0 0]	1.0087	0.0001
[1 1 1 0]	1.0122	0.0001
[0 0 1 0]	1.0096	0.0000
[0 1 1 1]	1.0100	0.0000
[0 0 0 1]	1.0062	0.0000
[1 0 1 0]	0.0034	0.0000
[1 0 1 1]	1.0094	0.0000
[1 1 0 1]	1.0086	0.0000
[0 0 0 0]	4.0288	0.0000
[1 0 0 0]	1.0081	0.0000
[1 1 1 1]	4.0326	0.0000
[0 1 0 1]	0.0005	0.0000

To evaluate the optimal asset combination within fifth group, comprising BE, CSIQ, SHLS, and SMR, the Variational Quantum Eigen solver (VQE), a hybrid quantum-classical algorithm, was employed. The VQE algorithm identified the optimal configuration as [1, 0, 0, 1], corresponding to the inclusion of BE and SMR, yielding the lowest objective function value of -0.0002. This configuration was also the most frequently observed outcome during the measurement process, with a dominant probability of 0.8099. This suggests high algorithmic confidence in this selection. Other configurations with non-negligible probabilities included [0, 0, 1, 1] (SHLS and SMR) with a value of 0.0012 and probability 0.0968, [1, 1, 0, 0] (BE and CSIQ) with a value of 0.0025 and probability 0.0772, [0, 1, 1, 0] (CSIQ and SHLS) with a value of 0.0039 and probability 0.0157.

Table 21. The Quantum Approximate Optimization Algorithm (QAOA) (Hybrid quantum-classical algorithm) of Fifth Group

Optimal: selection [1. 0. 0. 1.], value -0.0002		
----------------- Full result ---------------------		
selection	value	probability

[1 0 0 1]	**-0.0002**	**0.1672**
[0 1 0 1]	**0.0005**	**0.1671**
[0 0 1 1]	**0.0012**	**0.1668**
[1 1 0 0]	**0.0025**	**0.1665**
[1 0 1 0]	**0.0034**	**0.1663**
[0 1 1 0]	**0.0039**	**0.1661**
[0 0 0 1]	1.0062	0.0000
[0 0 0 0]	4.0288	0.0000
[1 0 0 0]	1.0081	0.0000
[1 1 1 1]	4.0326	0.0000
[0 1 0 0]	1.0087	0.0000
[1 1 1 0]	1.0122	0.0000
[0 0 1 0]	1.0096	0.0000
[0 1 1 1]	1.0100	0.0000
[1 0 1 1]	1.0094	0.0000
[1 1 0 1]	1.0086	0.0000

For Fifth Group, which includes the assets BE, CSIQ, SHLS, and SMR, the Quantum Approximate Optimization Algorithm (QAOA), a hybrid quantum-classical algorithm, was employed to search for the optimal asset selection. The result shows that the selection [1, 0, 0, 1], corresponding to choosing BE and SMR, achieved the optimal objective function value of -0.0002. This selection also exhibited the highest probability of occurrence at 0.1672 among all measured outcomes. Other configurations with similar probabilities include [0, 1, 0, 1] with a value of 0.0005 and a probability of 0.1671, [0, 0, 1, 1] with a value of 0.0012 and a probability of 0.1668, [1, 1, 0, 0] with a value of 0.0025 and a probability of 0.1665, [1, 0, 1, 0] with a value of 0.0034 and a probability of 0.1663, and [0, 1, 1, 0] with a value of 0.0039 and a probability of 0.1661. Each of these six selections had a probability of approximately 16.6%.

5 Conclusion

This study aimed to evaluate and compare the effectiveness of classical and quantum approaches in optimizing portfolios composed of leading global renewable energy stocks. By applying classical algorithms such as Advanced quantum-based methods including the Quantum Approximate Optimization Algorithm (QAOA) and Variational Quantum Eigen solver (VQE), we explored how different computational paradigms perform in real-world financial contexts. The findings demonstrate that both classical and quantum models can identify optimal portfolios with varying levels of risk and return. Quantum algorithms, particularly QAOA and VQE, were able to approximate optimal solutions with competitive performance to classical methods, even when run on simulated environments. Additionally, the clustering analysis using K-Means further assisted in grouping stocks with similar statistical properties, which served as a basis for more structured portfolio design. While classical methods remain reliable and computationally efficient, quantum approaches show promising potential, especially as quantum hardware and hybrid algorithms continue to evolve. This research contributes to a growing body of work at the intersection of quantum computing and financial optimization, highlighting that quantum techniques may become a viable complement or alternative to classical methods in portfolio construction.

Risk management is very challenging for portfolio optimization according to tail risk. The quantum distribution is a more precise measurement, especially more comprehensive than classical or normal distribution market behavior, particularly in the case of thin tails and fat tails phenomena (Zhang et al, (2016), Cunha et al, (2020), Cividino et al, (2023), and Ahn et al, (2024), Requião et al,(2024) and , Chukiat et al,(2024)). Therefore, these research results are useful for those who prefer to invest their money in the stock market more efficiently, especially in the renewable energy stock market.

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