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Tail Dependence in REITs Returns

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Overview

- Research Questions
- Existing Literature
- The Generic Models and Data Descriptions
- Results and Discussions
  - Marginal Distributions of REITs
  - Conditional Copulas
  - Tail Dependence of REITs with *t* Copula
- Conclusions

Research Questions

- Non-normality of asset return distribution (Mills (1927))

- Non-normality of asset returns *Joint* distribution
  - See e.g. Longin and Solnik (2001), Ang and Chen (2002)
  - A fall in values of assets beyond some thresholds can trigger a fall in values of other assets that are initially weakly correlated with the former

- Evidence of tail dependence and asymmetric dependence structure can be found in almost every market
  - International equity markets (Longin and Solnik (2001))
  - Equity and bond markets in G-5 countries (Hartmann et al. (2004))
  - Currency exchange markets (Patton (2006))
  - Asian developed future markets (Xu and Li (2009))
  - The US and European CDS markets (Coudert and Gex (2008))

Research Questions

- In Real Estate market,
  - Knight et al. (2005) – the potential benefits of diversification between real estate and equity markets during downturns disappears.

- Real Estate Boom-Burst
  - During the past ten years, real estate assets have tended to move closely together

- Tail Dependence between different sectors of real estate
  - Tail dependence
  - Asymmetry

Existing Literature

- Time-varying Conditional Copula (Patton (2002)), extending Copula (Sklar (1959))
  - The dependence structure of assets dynamically over time by conditional on the past information
  - Too many copulas to choose -> The copula with the highest loglikelihood value
- Patton (2006) finds the dependence between Deutsche Mark-USD and Yen-USD
  - highly asymmetric in one direction before the introduction of euro currency
  - marginally asymmetric in the other direction thereafter
- Jondeau and Rockinger (2006) investigate the dependence among major stock markets
  - Dependency between European markets increases over the sample period
- Goorah (2007) examines the dependence structure between US and UK listed real estate companies and REITs
  - Symmeterised Joe-Clayton (SJC) copula - best fit to the daily data from 1990Q1 to 2007Q1
  - Lower tail dependence is generally stronger than the upper one
- In our paper, we examine the tail dependence within US REITs

The Generic Model

- Copula-based AR, GJR-*t*-GARCH Model
  - The model consists of two components: Marginals and Copula
- Univariate AR's are used to capture the serial correlation
- Glosten-Jagannathan-Runkle(GJR)-t-GARCH can capture three features
  - Volatility clustering
  - Excess kurtosis of the return distribution
  - Leverage effect
- Conditional Copula to deal with (asymmetric) tail dependence
  - Gaussian, Student *t*, Gumbel, Frank, Clayton, SJC

The Generic Model

- Specification of the Marginal Distributions

$$r_{it} = \mu_{it} + \varepsilon_{it} = \bar{\mu}_i + \sum_{j=1}^{N} \sum_{\tau=1}^{d} \phi_{i,j,\tau} r_{j,t-\tau} + \varepsilon_{it}$$

$$\varepsilon_{it} = \sigma_{it} z_{it}$$

$$\sigma_{it}^2 = a_i + \sum_{\tau=1}^{p_i} b_{i,\tau} \sigma_{i,t-\tau}^2 + \sum_{\tau=1}^{q_i} c_{i,\tau} \varepsilon_{i,t-\tau}^2 + \gamma_i S_{i,t-1} \varepsilon_{i,t-1}^2$$

$$S_{t-1} = \begin{cases} 1 \ if \ \varepsilon_{t-1} < 0 \\ 0 \ if \ \varepsilon_{t-1} \geqslant 0 \end{cases}$$

$$\sqrt{\frac{\nu_i}{\nu_i - 2}} z_{it} \sim i.i.d.T_{\nu_i}$$

The Generic Model

- Specification of the Conditional Copula

$$\begin{aligned} \mathbf{z}_t &\sim i.i.d.\mathbf{F}_t(\mathbf{z} \,|\, \mathcal{F}_{t-1}) \\ &= \mathbf{C}\left(F_{1t}(z_1 \,|\mathcal{F}_{t-1}), F_{2t}(z_2 \,|\mathcal{F}_{t-1}), \ldots, F_{N,t}(z_N \,|\mathcal{F}_{t-1}) \,|\mathcal{F}_{t-1}, \boldsymbol{\alpha}_t\right) \\ &= \mathbf{C}\left(T_{\nu_1}\left(\sqrt{\frac{\nu_1}{\nu_1-2}} z_1 \,|\mathcal{F}_{t-1}\right), \ldots, T_{\nu_N}\left(\sqrt{\frac{\nu_N}{\nu_N-2}} z_N \,|\mathcal{F}_{t-1}\right) |\mathcal{F}_{t-1}, \boldsymbol{\alpha}_t\right) \end{aligned}$$

Copula	Type	Copula Parameter $(\boldsymbol{\alpha}_t)$
Gaussian	Constant	ρ_{ij}
	Time-varying	$\psi_{i,j,t} = \omega_{i,j,0} + \omega_{i,j,1} \cdot \psi_{i,j,t-1} + \omega_{i,j,2} \cdot \frac{\sum_{k=1}^{10} \Phi^{-1}(u_{i,t-k}) \cdot \Phi^{-1}(u_{j,t-k})}{10}$ $\rho_{i,j,t} = \tilde{\Lambda}(\psi_{i,j,t})$
Student t	Constant	ρ_{ij}, ν_{ij}
	Time-varying	$\psi_{i,j,t} = \omega_{i,j,0} + \omega_{i,j,1} \cdot \psi_{i,j,t-1} + \omega_{i,j,2} \cdot \frac{\sum_{k=1}^{10} \Phi^{-1}(u_{i,t-k}) \cdot \Phi^{-1}(u_{j,t-k})}{10}$ $\rho_{i,j,t} = \tilde{\Lambda}(\psi_{i,j,t})$ $\nu_{i,j,t} = \lambda_{i,j,0} + \lambda_{i,j,1} \cdot \nu_{i,j,t-1} + \lambda_{i,j,2} \cdot \tilde{\Lambda}^{-1}\left(\frac{\sum_{k=1}^{10} \lvert u_{i,t-k} - u_{j,t-k} \rvert}{10}\right)$

The Generic Model

- Specification of the Conditional Copula

Clayton	Constant	θ_{ij}
	Time-varying	$\theta_{i,j,t}=\lambda_{i,j,0}+\lambda_{i,j,1}\cdot\theta_{i,j,t-1}+\lambda_{i,j,2}/\left(-\log_2\left(1-\frac{\sum_{k=1}^{10}\lvert u_{i,t-k}-u_{j,t-k}\rvert}{10}\right)\right)$
Gumbel	Constant	θ_{ij}
	Time-varying	$\theta_{i,j,t}=\lambda_{i,j,0}+\lambda_{i,j,1}\cdot\theta_{i,j,t-1}+\lambda_{i,j,2}/\log_2\left(2-\left(1-\frac{\sum_{k=1}^{10}\lvert u_{i,t-k}-u_{j,t-k}\rvert}{10}\right)\right)$
Frank	Constant	θ_{ij}
	Time-varying	$\theta_{i,j,t}=\lambda_{i,j,0}+\lambda_{i,j,1}\cdot\theta_{i,j,t-1}+\lambda_{i,j,2}\cdot\frac{\sum_{k=1}^{10}\lvert u_{i,t-k}-u_{j,t-k}\rvert}{10}$
SJC	Constant	τ_{ij}^U,τ_{ij}^L
	Time-varying	$\psi_{i,j,t}^U=\lambda_{i,j,0}^U+\lambda_{i,j,1}^U\cdot\psi_{i,j,t-1}^U+\lambda_{i,j,2}^U\left(1/\log_2\left(2-\left(1-\frac{\sum_{k=1}^{10}\lvert u_{i,t-k}-u_{j,t-k}\rvert}{10}\right)\right)-1\right)$ $\psi_{i,j,t}^L=\lambda_{i,j,0}^L+\lambda_{i,j,1}^L\cdot\psi_{i,j,t-1}^L+\lambda_{i,j,2}^L/\left(-\log_2\left(1-\frac{\sum_{k=1}^{10}\lvert u_{i,t-k}-u_{j,t-k}\rvert}{10}\right)\right)$ $\tau_{i,j,t}^U=2-2^{1/(1+\psi_{i,j,t}^U)}$ and $\tau_{i,j,t}^L=2^{-1/\psi_{i,j,t}^L}$

The Generic Model

- The Estimation of Parameters : Two-step ML
  - Using the extension of Sklar theorem (Patton (2006))

$$\begin{aligned}\mathbf{f}_t(\mathbf{r}\,|\mathcal{F}_{t-1}) &= \frac{\partial^N}{\partial r_1\partial r_2\dots\partial r_N}\mathbf{C}_t\left(F_{1t}\left(r_1\,|\mathcal{F}_{t-1}\right),\dots,F_{N,t}\left(r_N\,|\mathcal{F}_{t-1}\right)|\mathcal{F}_{t-1}\right)\\ &= \prod_{i=1}^{N}\mathrm{f}_{it}\left(r_i\,|\mathcal{F}_{t-1}\right)\cdot\mathbf{c}_t\left(F_{1t}\left(r_1\,|\mathcal{F}_{t-1}\right),\dots,F_{N,t}\left(r_N\,|\mathcal{F}_{t-1}\right)|\mathcal{F}_{t-1}\right)\end{aligned}$$

  - Hence, the log-likelihood function can be expressed as

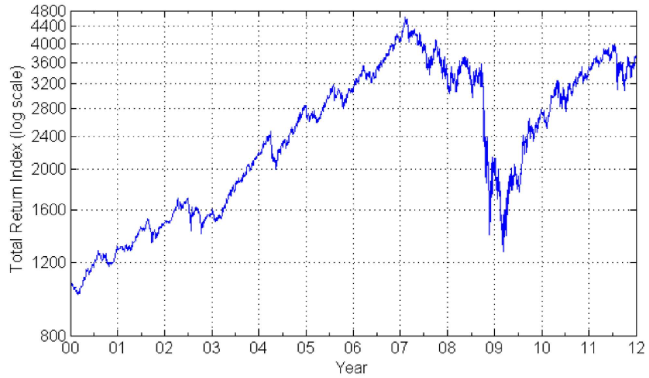
$$\begin{aligned}\log L\left(\boldsymbol{\theta},\boldsymbol{\alpha}_t\right) &= \sum_{t=1}^{T}\log\mathbf{f}_t(\mathbf{r}_t\,|\mathcal{F}_{t-1},\boldsymbol{\theta},\boldsymbol{\alpha}_t)\\ &= \sum_{i=1}^{N}\sum_{t=1}^{T}\log\mathrm{f}_i\left(r_{it}\,|\mathcal{F}_{t-1},\boldsymbol{\theta}_i\right)+\sum_{t=1}^{T}\log\mathbf{c}\left(U_{1t}\left(r_1;\boldsymbol{\theta}_1\right),\dots,U_{Nt}\left(r_N;\boldsymbol{\theta}_N\right)|\mathcal{F}_{t-1},\boldsymbol{\alpha}_t\right)\\ &= \sum_{i=1}^{N}\log L_i\left(\boldsymbol{\theta}_i\right)+\log L_c\left(\boldsymbol{\theta},\boldsymbol{\alpha}_t\right)\end{aligned}$$

Data Descriptions

- Daily returns of the U.S. equity REITs from 2000 to 2011:
  - Industrial & Office, Residential, Retail and Hotel & Lodging
- Total Return Indices of FTSE/NAREIT North America(Base date 1st Jan 2000 at 1,000)

Figure 1: Total return index of FTSE/NAREIT North America and total daily return for each sub-index

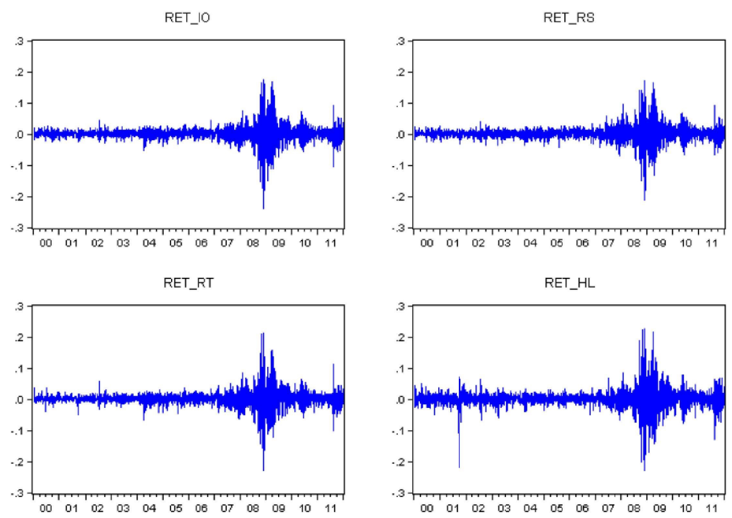
(a) The daily total return index of FTSE/NAREIT North American from 2000 to 2011



Data Descriptions

- Daily returns of the U.S. equity REITs from 2000 to 2011:
  - Industrial & Office, Residential, Retail and Hotel & Lodging

(b) The total daily returns for each sub-index: Industrial & Office (IO), Residential (RS), Retail (RT), and Hotel & Lodging (HL) REITs



Note: All the data are from DataStream. The y-axis represents unannualised daily continuously compounded total return, in which 0.3 means 30%. The x-axis is the two last digit of years.

Data Descriptions

Table 2: Descriptive statistics of data - US REITs sub-sector indices

	RET_IO	RET_RS	RET_RT	RET_HL
Mean ($\times 10^{-4}$)	3.44	4.81	5.27	3.42
Median ($\times 10^{-4}$)	4.42	4.73	5.22	5.96
Maximum ($\times 10^{-4}$)	1733.48	1693.21	2110.73	2262.53
Minimum ($\times 10^{-4}$)	-2383.46	-2129.46	-2298.03	-2298.07
Std. Dev. ($\times 10^{-4}$)	227.48	220.49	229.43	277.28
Skewness	-0.4093	-0.1160	-0.0094	-0.1516
Kurtosis	20.6155	17.8262	19.7049	17.8276
Jarque-Bera	40556.37	28674.76	36393.15	28685.05
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
ARCH F-Stat	99.8792	88.8541	100.5287	75.3971
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Augmented Dickey-Fuller Stat	-66.14	-67.71	-66.38	-65.39
	(0.0000)	(0.0000)	(0.0000)	(0.0000)

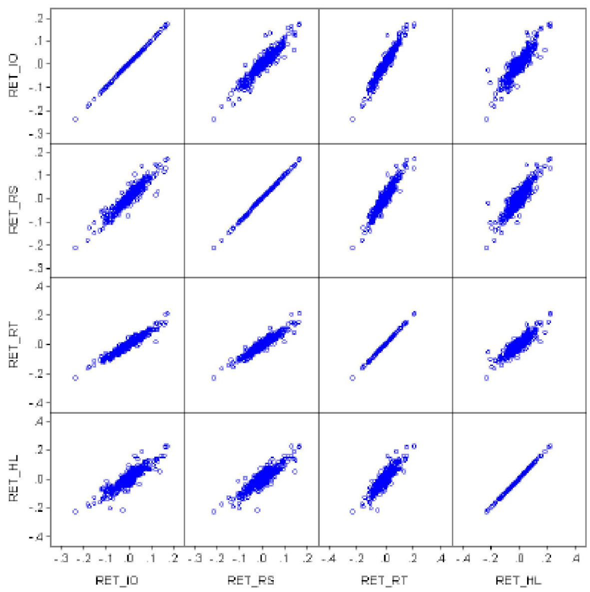
Correlation Matrix

	RET_IO	RET_RS	RET_RT	RET_HL
RET_IO	1.00	0.9293	0.9540	0.8612
RET_RS	0.9293	1.00	0.9355	0.8537
RET_RT	0.9540	0.9355	1.00	0.8596
RET_HL	0.8612	0.8537	0.8596	1.00

Note: The provided ARCH F-stats result from the test of an ARCH effect up to 20 lags. The ADF Stat reported is the one including an intercept and trend. The other two despite not being reported reject the unit root as well. The total number of observations is 3130. The value in parentheses are p-values.

Data Descriptions

Figure 2: Scatter plots of each pair of the REITs sub-index return series



Marginal Distributions of REITs

- AR terms become insignificant after including GARCH terms

- GARCH(1,2) for Residential REIT
- GARCH(2,1) for Industrial & Office REIT and Retail REIT
- GARCH(1,1) for Hotel & Lodging REIT

- In contrast to Cotter & Stevenson (2007)
  - Leverage effect is found for all four REITs
  - Industrial & Office REIT has the highest leverage effect

- Significant Student *t* degree of freedom
  - Implying the excess kurtosis of all four REITs

Marginal Distributions of REITs

Table 3: Estimation results: IO, RS, RT and HL REITs returns

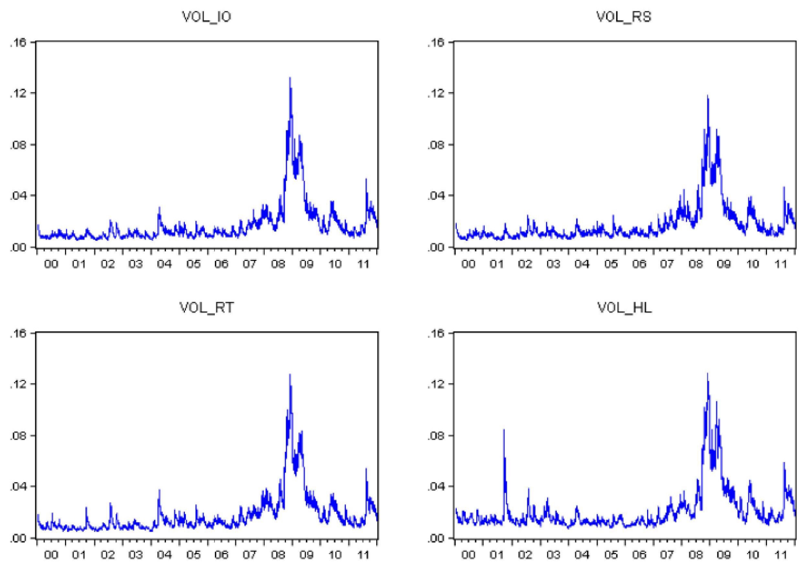
	RET_IO				RET_RS				RET_RT				RET_HL			
	Coeff	Std Err	z-Stat	Prob	Coeff	Std Err	z-Stat	Prob	Coeff	Std Err	z-Stat	Prob	Coeff.	Std. Err.	z-Stat	Prob
Mean Equation:																
INTC ($\times 10^{-2}$)	0.0913	0.018	5.098	0.000	0.0905	0.018	4.863	0.000	0.1117	0.018	6.304	0.000	0.0994	0.0241	4.125	0.000
Variance Equation:																
C ($\times 10^{-5}$)	0.235	0.067	3.525	0.000	0.117	0.038	3.100	0.002	0.219	0.063	3.463	0.001	0.466	0.112	4.146	0.000
ε^2_{t-1}	0.111	0.022	4.969	0.000	0.176	0.032	5.528	0.000	0.125	0.024	5.331	0.000	0.084	0.017	4.886	0.000
$\varepsilon^2_{t-1} \cdot 1_{\{\varepsilon_{t-1}<0\}}$	0.079	0.0229	2.738	0.006	0.030	0.016	1.817	0.069	0.064	0.028	2.239	0.025	0.058	0.021	2.680	0.007
ε^2_{t-2}	-	-	-	-	-0.106	0.031	-3.428	0.001	-	-	-	-	-	-	-	-
σ^2_{t-1}	0.404	0.149	2.706	0.007	0.914	0.010	89.708	0.000	0.392	0.135	2.910	0.004	0.088	0.013	70.031	0.000
σ^2_{t-2}	0.440	0.136	3.228	0.001	-	-	-	-	0.450	0.124	3.642	0.000	-	-	-	-
T-DIST. DOF	8.512	1.252	6.801	0.000	9.535	1.656	5.759	0.000	7.120	0.806	8.836	0.000	6.904	0.582	11.859	0.000

NOTE: The number of observations is 3130, which lies from 3/01/2000 to 30/12/2011. The error terms of each series are assumed to follow Student *t* distributions. The specification of the conditional mean of all series are constant. For the conditional variance, RET_IO and RET_RT follow GJR-t-GARCH(2,1), RET_RS follows GJR-t-GARCH(1,2), and lastly RET_HL follows GJR-t-GARCH(1,1).

Marginal Distributions of REITs

- Conditional Volatility for each error term of REITs returns

Figure 3: The conditional volatility from GARCH for each error term of REITs series



Conditional Copulas

Rank	Copula	LL $(\times 10^3)$	AIC	BIC
		IO vs RS		
1	TV Student t	2.563	-1.634	-1.623
2	TV Gaussian	2.486	-1.587	-1.581
3	Student t	2.449	-1.564	-1.560
4	TV SJC	2.373	-1.512	-1.501
5	TV Frank	2.361	-1.506	-1.501
6	TV Gumbel	2.355	-1.503	-1.497
7	Gaussian	2.330	-1.488	-1.486
8	SJC	2.299	-1.468	-1.464
9	Gumbel	2.2806	-1.4566	-1.4547
10	Frank	2.2699	-1.4498	-1.4478
11	TV Clayton	1.9912	-1.2704	-1.2646
12	Clayton	1.9308	-1.2331	-1.2312

Conditional Copulas

Rank	Copula	LL($\times 10^3$)	AIC	BIC
		IO vs RT		
1	TV Student t	2.893	-1.845	-1.833
2	TV Gaussian	2.841	-1.814	-1.808
3	TV SJC	2.679	-1.708	-1.696
4	TV Gumbel	2.673	-1.706	-1.700
5	Student t	2.636	-1.683	-1.679
6	TV Frank	2.610	-1.666	-1.660
7	Gaussian	2.511	-1.604	-1.602
8	SJC	2.481	-1.584	-1.580
9	Gumbel	2.437	-1.557	-1.555
10	Frank	2.428	-1.551	-1.549
11	TV Clayton	2.335	-1.490	-1.484
12	Clayton	2.132	-1.362	-1.360

Conditional Copulas

Rank	Copula	LL($\times 10^3$)	AIC	BIC
		IO vs HL		
1	TV Student t	1.452	-0.924	-0.912
2	TV Gaussian	1.416	-0.903	-0.897
3	TV SJC	1.359	-0.864	-0.853
4	TV Gumbel	1.322	-0.843	-0.837
5	TV Frank	1.301	-0.830	-0.824
6	Student t	1.259	-0.803	-0.800
7	SJC	1.200	-0.765	-0.762
8	Gaussian	1.186	-0.757	-0.755
9	Gumbel	1.164	-0.743	-0.741
10	Frank	1.157	-0.739	-0.737
11	TV Clayton	1.1084	-0.7063	-0.7005
12	Clayton	0.9679	-0.6178	-0.6159

Conditional Copulas

Rank	Copula	LL$(\times 10^3)$	AIC	BIC
		RS vs RT		
1	TV Student t	2.455	-1.565	-1.553
2	TV Gaussian	2.392	-1.526	-1.521
3	Student t	2.284	-1.458	-1.454
4	TV Gumbel	2.272	-1.450	-1.444
5	TV SJC	2.264	-1.443	-1.431
6	TV Frank	2.250	-1.436	-1.430
7	Gaussian	2.160	-1.379	-1.377
8	SJC	2.1459	-1.3699	-1.3661
9	Gumbel	2.1234	-1.3562	-1.3542
10	Frank	2.1116	-1.3486	-1.3467
11	TV Clayton	1.9105	-1.2188	-1.213
12	Clayton	1.7831	-1.1387	-1.1368

Conditional Copulas

Rank	Copula	LL$(\times 10^3)$	AIC	BIC
		RS vs HL		
1	TV Student t	1.352	-0.860	-0.849
2	TV Gaussian	1.322	-0.843	-0.837
3	TV SJC	1.257	-0.799	-0.788
4	TV Gumbel	1.211	-0.772	-0.766
5	TV Frank	1.205	-0.768	-0.762
6	Student t	1.178	-0.751	-0.747
7	SJC	1.1196	-0.7141	-0.7103
8	Gaussian	1.106	-0.7061	-0.7041
9	Gumbel	1.0868	-0.6938	-0.6919
10	Frank	1.0799	-0.6894	-0.6875
11	TV Clayton	1.0282	-0.6551	-0.6493
12	Clayton	0.8987	-0.5736	-0.5717

Conditional Copulas

Rank	Copula	LL($\times 10^3$)	AIC	BIC
		RT vs HL		
1	TV Student t	1.437	-0.914	-0.903
2	TV Gaussian	1.407	-0.897	-0.891
3	TV SJC	1.319	-0.839	-0.827
4	TV Gumbel	1.296	-0.826	-0.820
5	TV Frank	1.289	-0.822	-0.816
6	Student t	1.251	-0.798	-0.794
7	SJC	1.187	-0.757	-0.753
8	Gaussian	1.177	-0.751	-0.749
9	Frank	1.159	-0.740	-0.738
10	Gumbel	1.147	-0.732	-0.730
11	TV Clayton	1.107	-0.706	-0.700
12	Clayton	0.974	-0.622	-0.620

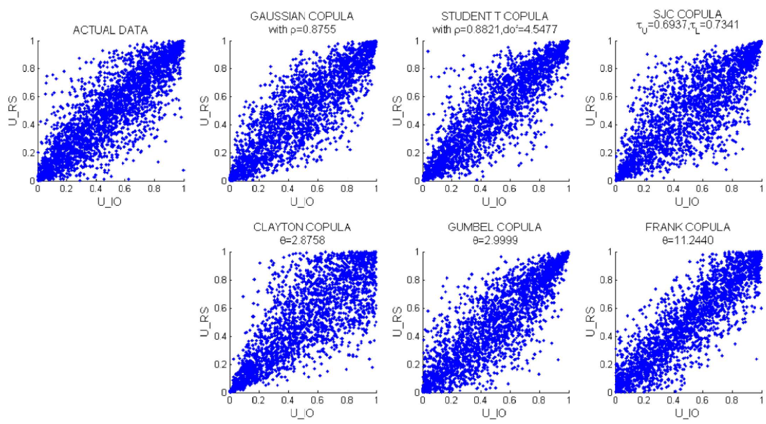
Conditional Copulas - Summary

- TV-Student *t* copula, which has symmetric tail dependence, provides the highest log-likelihood value, AIC, BIC in all cases.
- Hence, the pair-wise contemporaneous dependence structures of REITs
  - Not significantly asymmetric
  - Unlike other studies in various asset markets that find the asymmetric dependence structure
- The sample includes both the boom and burst in which REITs moved together closely.
- As REITs are highly correlated
  - The general shape of the dependence structure becomes more important

Conditional Copulas

- The shape of the dependence structure of REITs is closer to that of Student *t* and Guassian copulas.
- Scatter Plots of the Actual Probability Integral Value and the Simulated Value from Six Copulas: Residential REIT vs Industrial & Office REIT

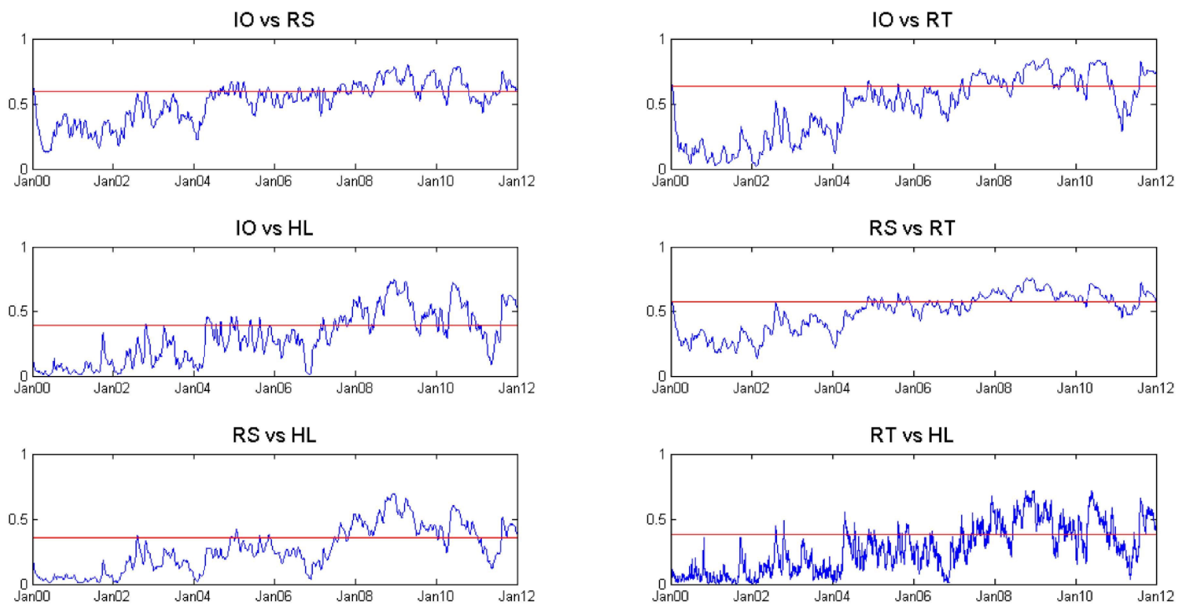
Figure 4: Scatter plots of the actual probability integral value and the simulated value from the six copulas: Industry & Office vs Residential REITs



NOTE: The number of simulated data is equal to the number of observations used in this study, i.e. 3130 observations.

Tail Dependence of REITs

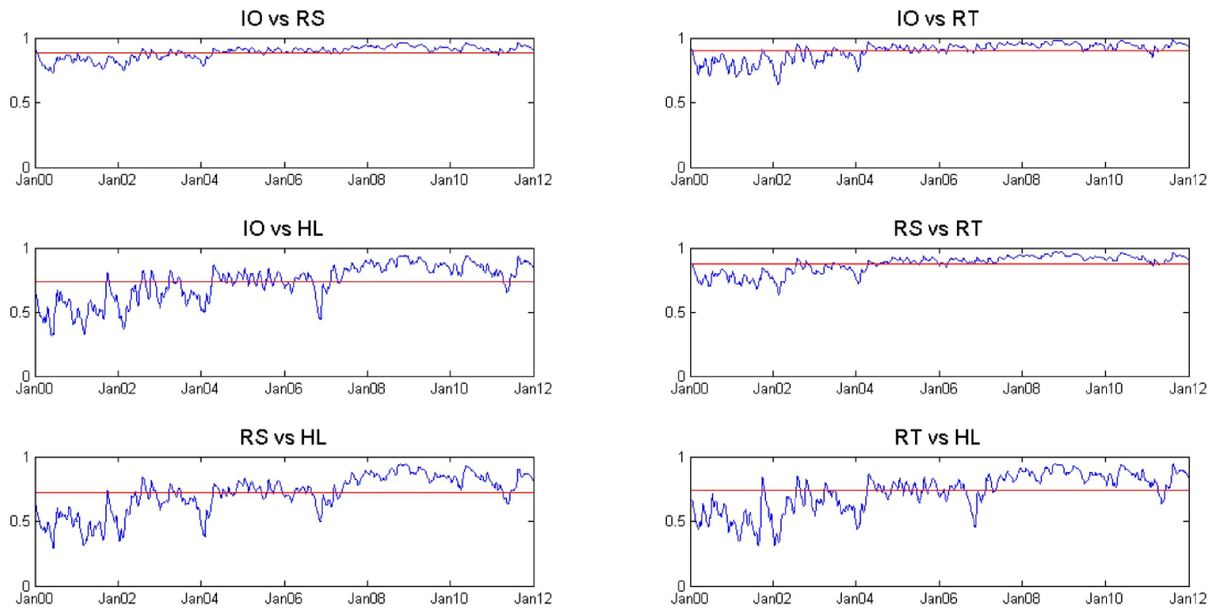
Figure 7: Time-varying contemporaneous tail dependence of REITs returns with Student t copula



Note: The horizontal line represents the corresponding constant parameters.

Linear Correlation of REITs

Figure 5: Time-varying linear correlation of REITs returns with Student t copula



Note: The horizontal line represents the corresponding constant parameters.

Conclusions

- Using daily data of four U.S. equity REITs (Jan 2000 - Dec 2011)
- Leverage effects found in every series
- Tail dependence within U.S. REITs is not significantly asymmetric
- The feature found in REITs over our sample period is the tail dependence during both the upturn and downturn.
  - However, the co-movements in downturn are clearly stronger due to leverage effects

Q & A

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